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SPHERICAL RAREFACTION FLOW IN THE NEIGHBOURHOOD OF A REFLECTION POINT OF A "BOUNDARY" CHARACTERISTIC†

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The structure of the one-dimensional steady spherically symmetric rarefaction flow of an ideal (inviscid and non-heat-conducting) gas in the neighbourhood of a reflection point of a "boundary" C^--characteristic is investigated in principal order. The "boundary" C^--characteristic separates the gas at rest from the flow due to the outward motion of a piston which confines the gas. In the rt plane, where r is the distance from the centre of symmetry and t is the time, the reflection point, which coincides with the point of arrival on the t axis of the boundary characteristic, coincides with the origin of coordinates. The initial velocity of the piston may be zero (for positive acceleration) or finite. When two symmetrical plane pistons advance, the "derived" derivatives of all the flow parameters on the C^--characteristic at the origin of coordinates, which in this case lies on the plane of symmetry, are finite. When a cylindrical and spherical piston advance, the derived derivative of the pressure (velocity) of the gas on the C^--characteristic at the origin of coordinates becomes minus (plus)-infinity although without intersecting characteristics of the same family [1–4]. © 2000 Elsevier Science Ltd. All rights reserved.

The infinite increase of the derivatives in the absolute value, irrespective of their sign, is sometimes regarded [5, 6] as evidence of a "gradient catastrophe", which, by destroying continuous flow, makes the isentropic expansion (and compression) of a gas "from rest to rest", introduced previously in [7] and later used in [8–10], impossible. The investigation carried out below, which is of independent interest, confirms the possibility of such a continuous isentropic process. A similar investigation was carried out previously [11] for cylindrically symmetric rarefaction flow.

1. BEHAVIOUR OF THE PARAMETERS IN THE NEIGHBOURHOOD OF THE BOUNDARY CHARACTERISTIC

Suppose p is the pressure, s and h are the specific entropy and enthalpy, $\rho = \rho(p, s)$ is the density, a is the velocity of sound and υ is the only (radial) component of the gas velocity. We will investigate the shock-free flows, beginning from the "uniform rest" state ($\upsilon \equiv 0, p \equiv p_0, s \equiv s_0$). By virtue of the equation of conservation of entropy in a particle and the initial homogeneity of the entropy $s \equiv s_0$ not only at the initial instant, the equations of continuity and motion [12] can be written in the form

$$\frac{\partial h}{\partial t}+\upsilon\frac{\partial h}{\partial r}+a^2\left(\frac{\partial \upsilon}{\partial r}+2\frac{\upsilon}{r}\right)=0,\quad \frac{\partial \upsilon}{\partial t}+\upsilon\frac{\partial \upsilon}{\partial r}+\frac{\partial h}{\partial r}=0,\quad a^{-2}=\rho_p=\left(\frac{\partial \rho}{\partial p}\right)_s \tag{1.1}$$

In addition to the trajectories of the particles, system (1.1) has two families of characteristics

$$\frac{\partial r}{\partial t}=\upsilon \pm a$$

where the upper (lower) sign corresponds to C^+ (C^-)-characteristics. As previously [3, 11], we will use the semi-characteristic variables $r\xi$ with ξ constant along the C^--characteristics. In these variables with the first two equations of system (1.1) and the equation for t take the form

$$h_r-a\upsilon_r+\frac{2a^2\upsilon}{(\upsilon-a)r}=0,\quad h_\xi+a\upsilon_\xi+(a-\upsilon)(\upsilon\upsilon_r+h_r)t_\xi=0,\quad t_r=\frac{1}{\upsilon-a} \tag{1.2}$$

$$\varphi_r=(\partial\varphi/\partial r)_\xi,\quad \varphi_\xi=(\partial\varphi/\partial\xi)_r$$

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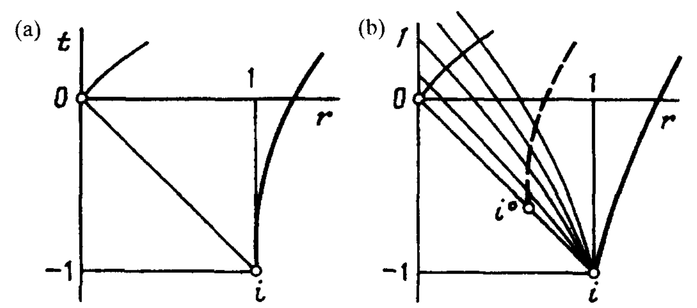

Fig. 1.

In Figs 1(a) and (b) the trajectories of the piston $r = R(t)$, which begins to move from the point i, are shown by the thick curves, while the characteristics are represented by the thin curves. The boundary C^--characteristic $i\,0$ separates the perturbed gas above it from the homogeneous gas at rest under it. We will give the parameters of the homogeneous gas at rest the subscript 0, and we will give quantities different from them at the point i the subscript i. In particular, $r_i = R(t_i)$ is the initial coordinate of the piston while $w_i = R''(t_i)$ is its initial acceleration. The rectilinear C^--characteristic $i\,0$ is the line of discontinuity of the derivatives t_ξ, υ_ξ, $h_\xi, \ldots$ for continuous υ, υ_r, $h_r, \ldots$ equal to zero. All the variables are dimensionless with r_i°, r_i°/a_i°, ρ_i°, $a_0^{\circ 2}$, $\rho_0^\circ a_0^{\circ 2}$ and $a_0^{\circ 2}/r_i^\circ$ as scales of length, time, velocity, density, specific enthalpy, pressure and acceleration of the piston. Dimensional quantities are given a superscript "degree". Corresponding to this we have $r_i = 1$, $t_i = -1$, $a_0 = \rho_0 = 1$. For an ideal gas $h_0 = 1/(\gamma - 1)$, while $p_0 = 1/\gamma$, where γ is the adiabatic index. Figure 1(a) corresponds to the motion of the piston with zero initial velocity and with initial acceleration $w_i > 0$, while Fig. 1(b) corresponds to the motion of the piston with finite initial velocity and with infinite acceleration. The second possibility is obtained from the first by taking the limit $w_i \to \infty$, while the first is obtained from the second by transferring in Fig. 1(b) from the piston trajectory with the beam of C^--characteristics at the point i to the trajectories of a particle which intersects the boundary characteristic at an arbitrary point i° with $0 < r_{i^\circ} < r_i = 1$.

When $t \geqslant 0$ a "reflected" C^+-characteristic emerges from the point 0, on which the derived derivatives are discontinuous. The velocity of the reflected characteristic at the point 0 is equal to $a_0 = 1$.

When $w_i < \infty$ in the region of the perturbed gas we put $\xi = t - t_i = t + 1$ at the point where the given C^--characteristic emerges from the piston trajectory. With this choice, the gas is at rest for $\xi < 0$ and is perturbed for $\xi > 0$, where

$$t_{\xi i} = 1, \quad \upsilon_{\xi i} \equiv w_i = R''(t_i) \tag{1.3}$$

Above the boundary characteristic (for $\xi = +0$), in view of the second equation of (1.2) and the method of normalization

$$h_\xi = -\upsilon_\xi \tag{1.4}$$

Differentiating the first equation of (1.2) with respect to ξ, and the second with respect to r, eliminating $h_{\xi r}$ from them and eliminating h_ξ using (1.4), we obtain $\upsilon_\xi = \upsilon_\xi/r$ for $\xi = +0$. Integrating this equation, taking the initial condition from (1.3) into account, we obtain

$$\upsilon_\xi = w_i/r \tag{1.5}$$

Similarly, an equation for t_ξ with $\xi = 0$ is obtained by differentiating the third equation of (1.2) with respect to ξ, taking into account the fact that $a = a(p, s)$ and $s \equiv s_0$. After eliminating h_ξ using (1.4) and substituting υ_ξ from (1.5), it takes the form

$$t_{\xi r} = -\alpha w_i / r; \quad \alpha = \omega_{pp0}/2, \quad \omega_{pp} = (\partial^2\omega/\partial p^2)_s \tag{1.6}$$

where $\omega = 1/\rho$ is the specific volume. For an ideal gas $\alpha = (\gamma + 1)/2$. Integration of Eq. (1.6), taking into account the condition for t_ξ from (1.3), gives the well-known formula [1, 3, 4]

$$t_\xi = 1 - \alpha\, w_i \ln r \tag{1.7}$$

According to relations (1.6) and (1.7) for a "normal" gas in which $\omega_{pp} > 0$, for outward motion of the piston ($w_i > 0$) the derivatives $h_\xi \to -\infty$, $\upsilon_\xi \to \infty$ where $r \to 0$, but t_ξ from (1.7) does not vanish on the boundary characteristic. Points in the rt plane, at which the Jacobian $D(r, t)/D(r, \xi) \equiv t_\xi = 0$, correspond to the intersection of C^--characteristics. Hence, as already noted, the gradient catastrophe does not occur on the boundary characteristic.

By virtue of (1.4), (1.5) and (1.7) on the boundary characteristic for $\xi = +0$ we have

$$\left(\frac{\partial h}{\partial t}\right)_r = -\left(\frac{\partial \upsilon}{\partial t}\right)_r = -\frac{\upsilon_\xi}{t_\xi} = \frac{-w_i}{r(1-\alpha w_i \ln r)} \approx \frac{1}{\alpha r \ln r} \tag{1.8}$$

The last equation holds in a fairly small neighbourhood of the origin of coordinates for values of r which satisfy the condition

$$r \ll r_* = \exp(-1/(\alpha w_i)) \tag{1.9}$$

A further investigation was carried out precisely for these values of r. In the case when the piston moves with a finite initial velocity ($\upsilon_i > 0$, $w_i = \infty$) in (1.9) $r_* = 1$. This is the maximum value of r_*.

When condition (1.9) is satisfied, expressions similar to the last formula in (1.8) hold on i 0 for the partial derivatives with respect to time of all the gas parameters, in particular, p and a. According to this, in the neighbourhood of the origin of coordinates, between the boundary characteristic and the reflection characteristic, i.e. when $-1 \leqslant \tau \equiv t/r \leqslant 1$ and for r satisfying condition (1.9), we will seek υ, h and a in "principal order" in the form

$$\upsilon(r,\tau) \approx \frac{V(\tau)}{\alpha \ln r}, \quad h(r,\tau) \approx h_0 + \frac{H(\tau)}{\alpha \ln r}, \quad a(r,\tau) \approx 1 + \frac{A(\tau)}{\alpha \ln r} \tag{1.10}$$

According to the definition, $\tau = -1$ on the boundary characteristic, where $\upsilon = 0$, $h = h_0$ and $a = 1$, and for r from (1.9) the "derived" partial derivatives of υ and h are given by the last equality in (1.8). Hence, by assuming that, in the principal order Eqs (1.10) allow of differentiation with respect to r and with respect to t, we obtain the following initial conditions for V and H

$$V(-1) = H(-1) = 0, \quad -V'(-1) = H'(-1) = 1 \tag{1.11}$$

Here and henceforth the prime denotes differentiation with respect to τ.

2. THE SOLUTION BETWEEN THE BOUNDARY CHARACTERISTIC AND THE TIME AXIS

By determining, from relations (1.10), the necessary partial derivatives and substituting them into Eqs (1.1), we obtain that the functions $V(\tau)$ and $H(\tau)$, when $-1 \leqslant \tau \leqslant 1$, obey the following equations

$$H' - \tau V' + 2V = 0, \quad V' - \tau H' = 0 \tag{2.1}$$

Substituting H' from the first equation of this system into the second, we arrive at an equation, the solution of which, satisfying the two conditions (for V and V') from (1.11), has the form $V(\tau) = (\tau^2 - 1)/2$. Substituting this expression into the first equation of system of (2.1) and integrating the equation thus obtained, taking into account the condition for H from (1.11), we find that $H(\tau) = \tau + 1$. The condition for H' from (1.11) is satisfied automatically as a consequence of the satisfaction of the corresponding conditions for V and V' and Eqs (2.1) for $\tau = -1$. The possibility of satisfying the two conditions from (1.11), when solving the first-order differential equation for V, is a consequence of the fact that for this equation the corresponding boundary characteristic of the point $\tau = -1$, $V = 0$ is a node. The point $\tau = 1$, $V = 0$, corresponding to the reflected characteristic, is also a node, but this is a result of the solution.

In view of the expressions obtained for V and H and the fact that the flow is isentropic in a neighbourhood of the origin of coordinates that is small in the sense of (1.9) when $-1 \leqslant \tau \leqslant 1$ for υ, h, p and a the following representations hold in principal order

$$\upsilon \approx \frac{\tau^2 - 1}{2\alpha \ln r}, \quad h \approx h_0 + \frac{\tau + 1}{\alpha \ln r}, \quad p \approx p_0 + \frac{\tau + 1}{\alpha \ln r}, \quad a \approx 1 + \frac{(\alpha - 1)(\tau + 1)}{\alpha \ln r} \tag{2.2}$$

Above the reflection characteristic solution (2.2) is incorrect because it gives infinite values of the parameters on the t-axis. The latter is clear because the condition of symmetry at the centre of the spherical volume filled with gas: $V = 0$ when $r = 0$, was not used to obtain it. By virtue of this condition the reflected characteristic for continuous parameters, as will be sèen below, will be the line of discontinuity of their derived derivatives. In order to take these features of the solution into account, in the region between the reflected characteristic and the t axis we will change from τ to $\eta = 1/\tau = r/t$, while the distributions of υ; h and a for $0 \leqslant \eta \leqslant r$ from (1.9) in principal order will be sought in the form

$$v(r,\eta) \approx \frac{V^\circ(\eta)}{\alpha \ln t}, \quad h(r,\eta) \approx h_0 + \frac{H^\circ(\eta)}{\alpha \ln t}, \quad a(r,\eta) \approx 1 + \frac{A^\circ(\eta)}{\alpha \ln t} \tag{2.3}$$

On the reflected characteristic $\eta = \tau = 1$ and $t = r$. Hence, by formulae (2.2) the parameters on it will be continuous if we require the following conditions to be satisfied

$$V^\circ(1) = 0, \quad H^\circ(1) = 2 \tag{2.4}$$

The continuity of the other thermodynamic parameters, other than the enthalpy, follows from the continuity of the enthalpy and the constancy of the entropy, when conditions (2.4) are satisfied. The corresponding linear relations between small increments in the enthalpy, pressure and velocity of sound (for constant entropy) have already been used when obtaining the formulae for p and a from (2.2).

Proceeding in the same way a when deriving system (2.2), we arrive at the equations

$$\eta \dot{V}^\circ - \eta^2 \dot{H}^\circ + 2V^\circ = 0, \quad \dot{H}^\circ - \eta \dot{V}^\circ = 0 \tag{2.5}$$

in which the dot denotes differentiation with respect to η.

Substituting $\dot{H}^\circ$ from the second equation of system (2.5) into the first, we arrive at an equation whose solution, which satisfies the first condition of (2.4), has the form $V^\circ(\eta) = C(\eta^2-1)/\eta^2$. The constant of integration C occurred in the formula for V° due to the fact that the point $\eta = 1$, $V^\circ = 0$ is a node. In view of the symmetry conditions mentioned above, $C = 0$ and $V^\circ \equiv 0$. Hence, in a small neighbourhood of the origin of coordinates above the reflected characteristic the gas is at rest "in principal order". Substituting $V^\circ \equiv 0$ into any of Eqs (2.5), we obtain $\dot{H}^\circ = 0$. Consequently, by virtue of the second condition from (2.4) $H^\circ \equiv 2$. Taking this into account we obtain (in the same way as (2.2)) that, in a small neighbourhood of the origin of coordinates in the sense of (1.9) above the reflected characteristic (when $0 \leqslant \eta \leqslant 1$), the following representations hold for υ, h, p and a in principal order

$$v \approx 0, \quad h \approx h_0 + \frac{2}{\alpha \ln t}, \quad p \approx p_0 + \frac{2}{\alpha \ln t}, \quad a \approx 1 + \frac{2(\alpha - 1)}{\alpha \ln t} \tag{2.6}$$

For fixed r the behaviour of the velocity of the C^+-characteristics $D = a + \upsilon$ depends on the quantity $\alpha = \omega_{pp0}/2$. If $\alpha > 1$, which, in particular, holds for an ideal gas, then, as follows from (2.2) and (2.6), D increases monotonically when the reflected characteristic intersects in the direction in which t increases. Consequently, the C^+-characteristics diverge here, ensuring continuity of the flow as in the problem of the outward motion of a cylindrical piston [11]. The difference from the cylindrical case is the fact that there the derived derivatives of D when $\tau = 1$ become infinite, in view of which the reflected rarefaction wave is "short". Here these derivatives are finite.

If we transfer from υ, p and the other parameters to their "normalized" increments: $\delta\varphi = (\varphi - \varphi_0)K_\varphi(\alpha)$ with $K_\upsilon(\alpha) = K_p(\alpha) = K_h(\alpha) = \alpha$ and with $K_a(\alpha) = \alpha/(\alpha - 1)$, then, as follows from (2.2) and (2.6), their fields will be independent of the gas "thermodynamics". According to results obtained previously in [11], in the neighbourhood of the "origin of coordinates" rarefaction flows with plane and cylindrical wave possess the same self-similarity. In them, however, the factors K_φ depend not only on α, but also on the initial acceleration of the piston w_i. The fact that the spherically symmetric rarefaction flow w_i in a small neighbourhood of the origin of coordinates is analogous to the self-similar Guderley solution on the collapse of spherical and cylindrical shock waves [13–15].

For inward motion with zero initial velocity and non-zero (negative) acceleration of a spherical piston the intersection of characteristics of the same family with the boundary characteristic and the formation on them of a shock wave moving towards the centre occurs for any $w_i < 0$, and for motion of a cylindrical piston the same occurs if $w_i < w_* < 0$, where w_{*i} is a certain "critical" value of the acceleration. For inward motion with a finite velocity extending to the centre or to the axis of symmetry, a shock wave, naturally, is formed immediately at the initial point of the piston trajectory. In all these cases the intensity of the shock wave formed increases without limit as $r \to 0$. Because of this flow in the neighbourhood of the point of arrival of the shock wave at the centre or on the axis of symmetry ($r = 0, t = t^S < 0$), containing a reflected shock wave, is described by the above-mentioned self-similar solution, irrespective of the "prehistory" (of the values of υ_i and w_i). As also in the spherically symmetric rarefaction flow considered, in these compression flows the prehistory only governs the values of r and t—the dimensions of the region of self-similarity in the neighbourhood of the point $r = 0, t = t^8$. In this case compression flows corresponding to special piston trajectories will be exceptional (non-self-similar down to $r = 0$, it is true, not as regards the increase in the intensity of the shock wave but as regards the distributions of the parameters after it). The latter should ensure focusing at the same point: $r = 0, t = t^S$ of a beam of the C^--characteristics that overtake the shock wave.

3. EVOLUTION OF THE SOLUTIONS WHEN THERE IS A CHANGE IN THE TYPE OF SYMMETRY

Using formulae (2.2) and (2.6) and similar results, obtained for the cases of plane and cylindrical symmetry [11], we will compare the distributions $\delta\upsilon$ and δp in a small neighbourhood of the origin of coordinates for the motion of plane, cylindrical and spherical pistons.

In the case of the symmetrical motion of two plane pistons in this neighbourhood with corresponding introduced normalizing factors K_υ and K_p—the functions α and w_i in principal order we have [11]

$$\delta p \approx (1+\tau)r = r+t, \quad \delta v \approx -(1+\tau)r = -(r+t) \quad (-1 \leqslant \tau \leqslant 1)$$
$$\delta p \approx 2t, \quad \delta v \approx -2r \quad (0 \leqslant \eta \leqslant 1) \tag{3.1}$$

Similarly, rewriting the results previously obtained [11] for the motion of a cylindrical piston in the notation used above, we will have

$$\delta p \approx P(\tau)\sqrt{r}, \quad \delta v \approx -V(\tau)\sqrt{r} \quad (-1 \leqslant \tau \leqslant 1)$$
$$\delta p \approx P^\circ(\eta)\sqrt{t}, \quad \delta v \approx -\eta V^\circ(\eta)\sqrt{t} \quad (0 \leqslant \eta \leqslant 1) \tag{3.2}$$

with the functions $P(\tau), \ldots$ from [11], represented in Fig. 2. Here, in the second formula for $\delta\upsilon$ in (3.2) the error which occurs in [11] in introducing but not when calculating $V^\circ(\eta)$ has been corrected. In Fig. 2, curves 1 and 2 show the functions $P(\tau)$ and $V(\tau)$, while curves 1° and 2° are the functions $P^\circ(\eta)$ and $V^\circ(\eta)$.

In the neighbourhood of the origin of coordinates the pressure and velocity isolines corresponding to the plane case, by virtue of formulae (3.1), are parallel straight lines which suffer a discontinuity on

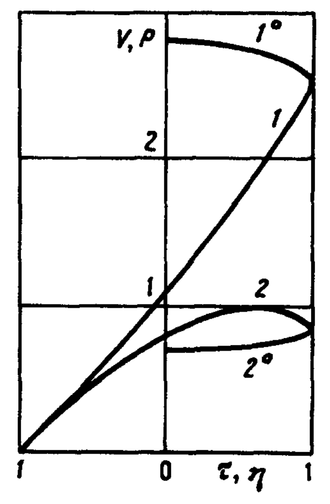

Fig. 2.

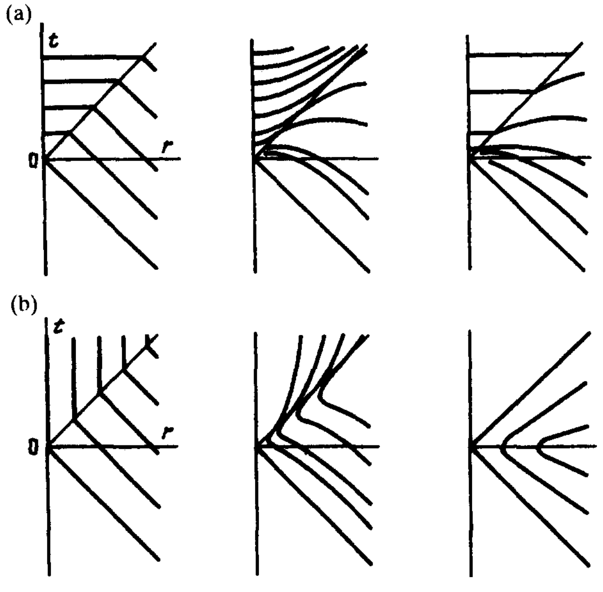

Fig. 3.

the reflected characteristic. They are shown in the left patterns of Figs 3(a) and (b), respectively. In Fig. 3 the isolines, including the boundary characteristic and the upper part of the ordinate axis, are represented by the bold continuous line while the reflected characteristic is represented by the fine straight line.

According to formulae (3.2) for δp for the motion of a cylindrical piston in the neighbourhood of the origin of coordinates on the isobars

$$\frac{dt}{dr}=\frac{2\tau P'-P}{2P'}\quad(-1\leqslant\tau\leqslant 1),\qquad \frac{dt}{dr}=\frac{2\dot{P}^\circ}{2\eta\dot{P}^\circ-P^\circ}\quad(0\leqslant\eta\leqslant 1)$$

Since the right-hand sides of these equations are functions of only τ or η, all the isobars intersect each ray, emerging from the origin of coordinates, at the same angle. This property is taken into account qualitatively in the middle pattern of Fig. 3(a), constructed taking the behaviour of curves 1 and 1° of Fig. 2 into account.

Similarly, by virtue of formulae (3.2) on the lines $\delta \upsilon$ = const we have

$$\frac{dt}{dr}=\frac{2\tau V'-V}{2V'}\quad(-1\leqslant\tau\leqslant 1),\qquad \frac{dt}{dr}=\frac{2(V^\circ+\eta\dot{V}^\circ)}{\eta(V^\circ+2\eta\dot{V}^\circ)}\quad(0\leqslant\eta\leqslant 1)$$

The velocity isolines, constructed taking into account the behaviour of curves 2 and 2° (Fig. 2), are shown in the middle pattern of Fig. 3(b). According to Fig. 2 and the expressions obtained previously [11], when $\tau = \eta = 1$ the derivatives in the formulae obtained above for the slope of the isobars and the constant-velocity lines become infinite. Hence, the isolines shown in the middle patterns of Fig. 3 intersect the reflected characteristic tangential to it.

By (2.2) for the motion of a spherical piston in the neighbourhood of the origin of coordinates when $-1 \leqslant \tau \leqslant 1$ along an isobar

$$\frac{dt}{dr}=\tau+\frac{\tau+1}{\ln r}$$

In a small neighbourhood of the origin of coordinates $\ln r < 0$ and $|\ln r| \gg 1$. Hence, in principal order, under the reflected characteristic and on approaching it from below, the slope of an isobar is slightly less (taking the sign into account) than the slope of the ray $t = \tau r$, arriving at the same point. In the same order, above the reflected characteristic, by virtue of (2.6), the isobars are sections of straight lines parallel to the abscissa axis. The pattern on the right of Fig. 3(a) qualitatively shows these features of the isobars of spherically symmetric rarefaction flow.

Similarly, by virtue of (2.2), along the velocity isolines in the motion of a spherical piston for $-1 \leqslant \tau \leqslant 1$, i.e. between the boundary characteristic and the reflected characteristic

$$\frac{dt}{dr} = \tau + \frac{\tau^2 - 1}{2\tau \ln r}$$

In view of this formula, the slope of the velocity isolines above the r axis, where $(\tau^2-1)/\tau < 0$, is greater than, while under the r axis, where $(\tau^2-1)/\tau > 0$, it is less than the slope of the rays $t = \tau r$. The slopes of the velocity isolines and of the rays $t = \tau r$ differ considerably only in a small neighbourhood of the abscissa axis, on approaching which the slope of the isolines approaches infinity. The right pattern in Fig. 3(b) qualitatively reflects these features of the velocity isolines. We recall that, according to (2.6), $\delta\upsilon = 0$ in principal order above the reflected characteristic.

With the obvious difference between the rarefaction flow isolines with plane, cylindrical and spherical symmetry, their evolution on passing from the left to the right patterns of Fig. 3, *a* and *b* is natural. This is important because the distributions of the parameters obtained above and in [11] are based on the assumption that representations (1.10), (2.3) and their analogues in [11] are differentiable. In the case of cylindrically symmetric flow the results obtained in [11] are confirmed by the analysis of the solution of the inverse problem carried out in [16]. For values of δp specified on the semi-axis $t > 0$, by (3.2) when $\delta p = 0$ for $t < 0$ the properties of the solutions of the direct problem [11] and the inverse problem [16] are identical.

Thus, for the given distribution of δp in [16], as in [11], in the first approximation on the reflected characteristic the derived derivatives, being continuous, have a logarithmic singularity. In the next iteration it is "quenched" by the same singularity of the derivatives of the velocity of the C^+-characteristics D. At the same time, the spherically symmetry inverse problem, investigated in [16], with a power distribution of p, specified *a priori* on the semi-axis $t > 0$, by virtue of (2.6) differs essentially from the direct problem which occurs in the outward motion of a piston.

4. CONCLUSION

The unlimited increase in the derivatives of the parameters of spherically symmetric and cylindrically symmetry flows as one approaches the reflection point of the boundary characteristic from the t-axis is a property which, in general, is inherent not only in this but any "discontinuous" C^--characteristic. Here the term "discontinuous" applies only to a characteristic which emerges from a point of the piston trajectory at which its acceleration or velocity breaks, or the characteristic, produced by reflection from the piston trajectory of the C^+-characteristic, which carries discontinuities of the first or second derivatives of the parameters. It is obvious that such discontinuous C^--characteristics are, in this sense, exceptional, unlike the continuum of C^--characteristics of the rarefaction flow which arises when flow occurs around the initial (accelerating) part of the piston trajectory, and for a finite initial piston velocity—a centred rarefaction wave. Along these, the derivatives of all the parameters, as one approaches the t-axis, naturally remain finite. In the problem of isentropic rarefaction or compression from rest to rest (the second) is obtained from the first by changing the signs of the velocity and time) the total derivatives along the rectilinear closing C^+-characteristic are zero everywhere, including the point where it emerges from the t-axis. Hence it follows immediately that, at this point, the derivatives are finite both below and above the closing C^--characteristic (with respect to the first derived derivative) is ensured by a special choice of the piston trajectory, which is found from the solution of the corresponding Goursat problem.

Spherically symmetric rarefaction and compression from rest to rest are of interest not only in themselves but also as one of the ways of solving the problem, discussed in [6], of the possibility of such a transition for an arbitrary three-dimensional volume. To set up this transition it is sufficient to describe the initial spatial volume, bounded by impenetrable walls, by a sphere, then considered as a spherical piston. For the law of motion of the walls of an ideally flexible spatial piston, which, in accordance with the calculated spherically symmetric flow, it is convenient to determine using the Lagrange variable [12], the final form of the volume obtained will be similar to the initial form.

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